

Roll No.

Total Pages : 03

BT-1/D-22

41040

MULTI-VARIABLE CALCULUS AND
LINEAR ALGEBRA
BS-135A

Time : Three Hours]

[Maximum Marks : 75

Note : Attempt *Five* questions in all, selecting at least *one* question from each Unit.

Unit I

1. (a) Prove the following :

7.5

$$\int_0^{\infty} \frac{e^{-x^2}}{\sqrt{x}} dx \times \int_0^{\infty} x^2 e^{-x^4} dx = \frac{\pi}{4\sqrt{2}}.$$

- (b) Find the volume of the solid generated by revolution of the plane area bounded by $y^2 = 16x$ and $y = 4x$ about the x -axis.

7.5

2. (a) Discuss the applicability of Rolle's theorem to the function $f(x) = |x|$ in $[-1, 1]$.

7.5

- (b) Evaluate the limit of the function :

7.5

$$\lim_{x \rightarrow 0} (\operatorname{cosec} x)^{\frac{1}{\log x}}.$$

Unit II

3. (a) Test the convergence of the $\sum \frac{n^3 + a}{2^n + a}$. 7.5

(b) Discuss the convergence of the series : 7.5

$$\frac{1^2}{2^2} + \frac{1^2}{2^2} \cdot \frac{3^2}{4^2} + \frac{1^2}{2^2} \cdot \frac{3^2}{4^2} \cdot \frac{5^2}{6^2} + \dots$$

4. (a) Find the Fourier series to represent : 7.5

$$f(x) = \begin{cases} 1 + \frac{2x}{\pi}, & -\pi \leq x \leq 0 \\ 1 - \frac{2x}{\pi}, & 0 \leq x \leq \pi \end{cases}$$

(b) Obtain the Fourier series of the function : 7.5

$$f(x) = \begin{cases} \pi x, & 0 \leq x \leq 1 \\ \pi(2-x), & 1 \leq x \leq 2. \end{cases}$$

Unit III

5. (a) Expand $\cos x$ in powers of $\left(x - \frac{\pi}{2}\right)$, and hence find the value of $\cos 91^\circ$ corrected up to four decimal places. 7.5

(b) If $u = f(r)$ where $r^2 = x^2 + y^2$, prove that : 7.5

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r).$$

6. (a) If $(\cos x)^y = (\sin y)^x$, find $\frac{dy}{dx}$. 7.5

(b) A thin closed rectangular box is to have one edge equal to twice the other, and a constant volume 72 m^3 . Find the least surface area of the box. 7.5

Unit IV

7. (a) By applying the Gauss Jordan Method, find the

inverse of the matrix $\begin{bmatrix} -1 & 0 & 6 \\ 3 & 6 & 1 \\ -5 & 1 & 3 \end{bmatrix}$. 7.5

(b) If matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$, find the matrix

represented by : 7.5

$$A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 6A^2 - 3A + 2I.$$

8. Find the eigen values and the corresponding eigen vectors

of the matrix given by $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$. 15